Semester One

ATAR course examination, 2023

Question/Answer Booklet

MATHEMATICS
SPECIALIST - UNIT 1

Section One:
Calculator-free

WA Student Number: In figures

In words   _____

Student Name   _____

Time allowed for this paper
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this paper
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	10	10	100	92	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2023: Part II Examinations.* Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator Free   **(51 marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1   (5 marks)

49 is called 'prime-looking' because it is composite but not divisible by 2, 3 or 5.
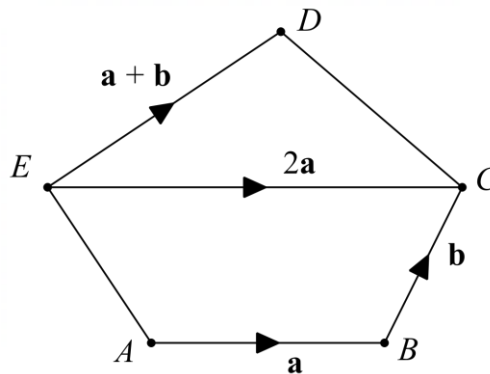
49 is the first prime-looking number.

There are 25 prime numbers between 1 and 100 inclusive.

Use include-exclusion principle to show that there are only 3 **prime-looking** numbers between 1 and 100 inclusive.

Question 2 **(6 marks)**

$ABCDE$ is a pentagon, with $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$, $\overrightarrow{EC} = 2\mathbf{a}$ and $\overrightarrow{ED} = \mathbf{a} + \mathbf{b}$.

(a)   Determine, in terms of $\mathbf{a}$ and $\mathbf{b}$,

   (i)   an expression for $\overrightarrow{CD}$. (2 marks)

   (ii)   an expression for $\overrightarrow{AE}$. (2 marks)

(b)   Hence, state two geometric facts about the sides CD and AE. (2 marks)

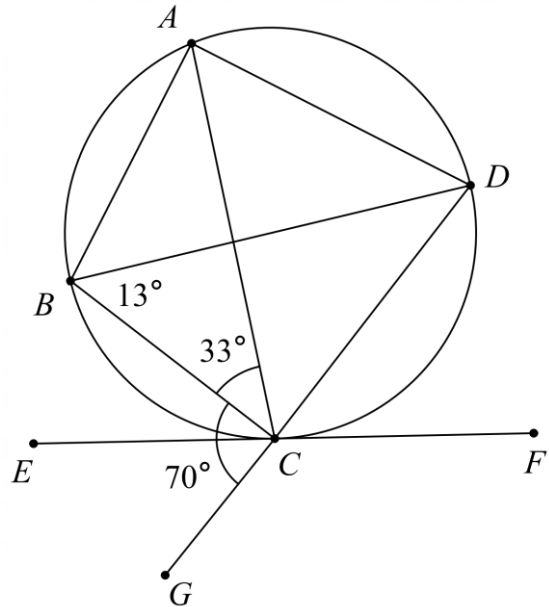**Question 3** **(8 marks)**

$ABCD$ is a cyclic quadrilateral.

EF is a tangent at C, and DCG is a straight line.

$\angle BCA = 33°$
$\angle GCB = 70°$
$\angle DBC = 13°$

(a) Determine the following angles, giving reasons

    (i) $\angle BAD$ (2 marks)

    (ii) $\angle BDA$ (2 marks)

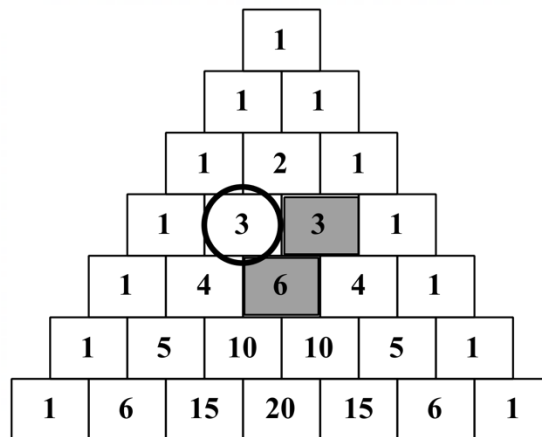(b) Prove that AC passes through the centre of the circle, justifying your answer.   (4 marks)

Question 4   **(6 marks)**

The first seven rows of Pascal's triangle are given below.

In this example, the square of the circled number, is equal to sum of the two shaded numbers i.e. $3^2 = 3 + 6$.

(a)   Write down the two numbers in Pascal's triangle that have a sum of 49.   (2 marks)

(b)   Using combinations prove this pattern holds for any second element of any row.   (4 marks)

               See next page

Question 5 **(9 marks)**

(a) Given $^nP_r = 90$ and $^nC_r = 45$, determine the value of n and the value of r. (4 marks)

(b) (i) Show that the number of different 9-letter arrangements of the word 'TENNESSEE' is $\frac{3}{4} \times 7!$. (2 marks)

(ii) Lara believes that the number of different 11-letter arrangements of the word 'MISSISSIPPI' is greater than of the arrangements of the word 'TENNESSEE'. Is she correct? Justify your answer mathematically. (3 marks)

Question 6 **(9 marks)**

(a) A parallelogram $ABCD$ is defined by vectors $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}.$

If $\mathbf{a} = (x - 1)\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + (x^2 - 1)\mathbf{j}$, determine the value(s) of x, such that $ABCD$ is a rectangle. (3 marks)

(b) The angle between two vectors, $\mathbf{c}$ and $\mathbf{d}$ is $60°$, with $|\mathbf{c}| = 4$, $|2\mathbf{c} - \mathbf{d}| = 7$ and $|\mathbf{c}| < |\mathbf{d}|$.

(i) Show, with aid of a diagram, that $|\mathbf{d}| = 5$. (3 marks)

(ii) Hence, or otherwise, determine the length of $\mathbf{c} + 2\mathbf{d}$. (3 marks)

See next page

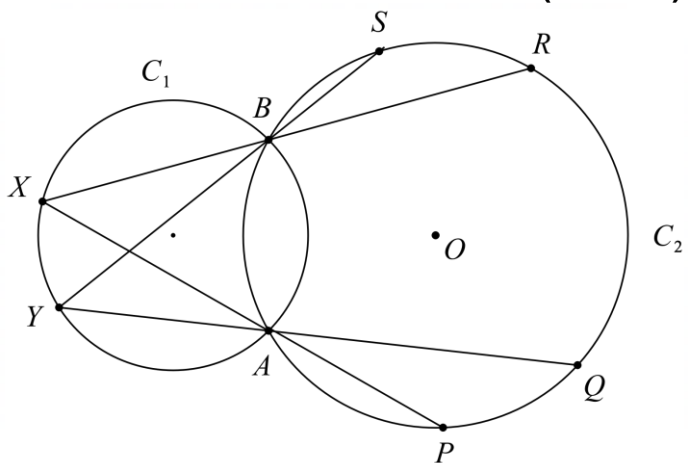**Question 7** **(8 marks)**

Two circles, C_1 and C_2, intersect at A and B.

Two points X and Y are on C_1.

The line XA extended intersects with C_2 at P, and YB extended intersects with C_2 at S.

The line XB extended intersects with C_2 at R, and YA extended intersects with C_2 at Q.

O is the centre of C_2.

(a)   Prove that $\angle PAQ = \angle SBR$, giving reasons. (3 marks)

(b)   Prove the chords PR and QS are congruent. (5 marks)

End of Calculator Free Section